BACK

CAN PROCEDURAL AND CONCEPTUAL MATHEMATICAL KNOWLEDGE BE LINKED THROUGH COMPUTER ASSISTED LEARNING?#

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Abstract
Despite its high educational relevance, just a few CAL studies have examined secondarily whether their computer-based treatments related procedural and conceptual mathematical knowledge. This study summarizes their findings, explains how these knowledge types may be linked, and proposes a constructivist CAL environment enabling these knowledge types to be connected.

INTRODUCTION

According to Fey (1989), computers can be used to change mathematics teaching by decreasing the time needed for procedural skills and increasing the time for conceptual understanding, the importance of which has been realized by many researchers (see, for example, Grouws, 1992). If we agree that a main goal of mathematics education is to develop both procedural and conceptual knowledge and make links between the two, a very important research question regarding computer-based mathematics education is "how different technologies affect the relation between procedural and conceptual knowledge" (Kaput, 1992; p. 549). However, just a few CAL studies have examined the effects of their treatments regarding the coordination of procedural and conceptual mathematical knowledge. While Schwarz et al. (1990) and Simmons & Cope (1997) evidenced that the links between these knowledge types (the P-C links) can be established, Yerushalmy (1991), Hochfelsner & Kligner (1998) and

A shortened report of a study realized with Lenni Haapasalo, University of Joensuu, Finland. This study is available at http://www.mi.sanu.ac.yu/~djkadij/acdca6.htm

Laborde (2000) found that their treatments did not promote the P-C links. It is important to underline that most of these studies neither clearly define the relevant notions regarding procedural and conceptual knowledge measuring them reliably, nor thoughtfully examine the question of the P-C links at the theoretical and the instructional levels. In the remaining text the latter issues will be briefly examined by assuming a knowledge distinction proposed by Haapasalo & Kadijević (2000).

HOW MAY THESE KNOWLEDGE TYPES BE LINKED?

According to Haapasalo & Kadijević (2000), an answer to this question depends whether one assumes the reliance of conceptual knowledge on procedural knowledge or vice versa.[1]

Many researchers find that procedural knowledge enables conceptual knowledge development. An instructional implication is: Utilize procedural knowledge and reflect on the outcome. Having assumed this knowledge reliance, the P-C links may be promoted through microworlds coordination (Papert, 1987) or proceptual thinking (Gray & Tall, 1993).

Most, perhaps the majority of, researchers/educators assume that conceptual knowledge enables procedural knowledge development. An instructional implication is: Build meaning for procedural knowledge before mastering it. Having presupposed such a knowledge reliance, it is utilization competence (Gelman & Meck, 1986) or production rules utilization (Anderson, 1983) whereby the P-C links may be promoted.

[1] According to Vygotsky (1978), procedural knowledge does precede conceptual know-ledge ontogenetically, but it is school learning that precedes intellectual development. While the former dependence, for example, may be suitable for introducing the conceptof a limit that promotes its dynamic definition, the latter one may be appropriate for teaching fractions and decimals (Haapasalo & Kadijević, 2000).

CAL ENVIRONMENT PROMOTING THE P-C LINKS

According to Anderson (1983), the P-C links can be promoted through learning activities requiring production rules utilization[2] . Such activities may be realized within LISD (Learning through Intelligent Software Development) - an approach to learning mathematics through knowledge engineering (Kadijević, 1998, 1999, 2000). As this approach utilizes, among others, programming in logic and *PROLOG* enabling the representation and flexible use of procedural and declarative (conceptual) knowledge, it is reasonable to expect that the LISD treatment would not only promote the acquisition of both procedural and conceptual knowledge, but also relate these knowledge types. A recent study (Kadijević, 1994) gave some empirical evidence for such a claim. By applying a pre-test/post-test design[3], a significant correlation was found between procedural knowledge scores and conceptual knowledge scores on the post-test, which was not present on the pre-test. However, more empirical evidence regarding the P-C links within the LISD or a LISD-like treatment is still needed. Having in mind the relevance of the affective domain to problem solving performance (e.g., Schoenfeld, 1992), further research may examine the P-C links not only in cognitive, but also in affective terms.

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[2] Although Anderson (1983) postulated that procedural knowledge is represented by production rules, the LISD production rules frequently deal with both knowledge types. For fuller detail see, for example, Kadijević (1999).

[3] A description of the applied method can be found in Kadijević (1998); the basic facts regarding the applied tests can be found at http://www.mi.sanu.ac.yu/~djkadij/tests.htm

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