BACK

POTENTIALS AND EFFECTS OF 'REPRESENTING' IN CAS-SUPPORTED MATHEMATICS TEACHING

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Abstract
The multitude of different (forms of) representation(s) is most frequently mentioned as didactic advantage of CAS. Special terms such as 'multiple linked representation' or 'window-shuttle-principle' are used, numerous teaching examples are offered (cf. e. g. Aspetsberger 1997; Berry et al. 1994; Canet 1996; Heugl et al. 1996).
In this paper the offer of representations in a CAS-supported mathematics classroom as well as their didactical potentials and effects are analysed.

1. Initial Remarks

The question of didactical advantages, potentials and effects cannot be discussed and evaluated in a sensible way regardless of a (theoretical) frame of reference. The 'educational philosophy' of a modern generally educating mathematics classroom discussed in the paper of W. Peschek and E. Schneider (in this volume) represents the frame of orientation for the analysis in the following items. The (theoretical) discussions of didactical advantages of different (forms of) representation(s) in the sense of this 'philosophy of education' from a social and an individual point of view (above all expanding the ability to communicate; supporting the (individual) development of concept-meaning) base especially on considerations of R. Fischer (1984) and H. Steinbring (1999) explained more detailed within the lecture.

2 CAS and Forms of Representation

CAS offer schematic as well as symbolic forms of representation (cf. Fischer (1984) for this classification of representations):

On *schematic level* CAS 'know' graphical and (in most cases) tabular representations, on *symbolic level* there are arithmetic and algebraic representations available and CAS support also verbal representations in limited manner (e. g. comments, scripts).

The offer of CAS-forms of representation does not essentially differ from the offer of 'hand-drawn representations'. In order to materialize concepts different forms of representation were also used without CAS (e. g. teaching materials; schoolbooks). But, by using CAS significant facilities and simplifications are brought into the practical and flexible availability of different forms of representation. It is for the first time becoming really *'efficient' to use different forms of representation*:

- CAS construct *quite rapidly* and *without much effort* different representations of (symbolically materialized) concepts.
- CAS enable and support a rapid *switching between the different forms of representation.*
- The small amount of effort necessary for producing CAS-representations supports *changes* and *manipulations of CAS-representations.*
- CAS *make representations accessible* which could manually not be used in an adequate way because of the great amount of operative effort (e. g. recursive representations, 3D-representations, modules).

3. Shifting of Meanings of Representations by CAS

One of the consequences of the simple and rapid construction of different (forms of) representation(s) is the shifting of meanings of some representations. I will illustrate this on the basis of tables.

The meaning of tables - without CAS - lies primarily in their role as an tool for constructing function graphs; they also allow the easy 'reading' of function values. With CAS function graphs are constructed 'by pressing a button'; tables are not necessary for the construction. CAS determine any function value required 'by pressing a button'; it is not necessary to gather these informations from tables. That means, on the one hand, tables can be produced by CAS without any great effort 'by pressing a button' and are, therefore easily accessible to the students; on the other hand they turn renouncable in their previous role! So does handling with tables make any sense any more? Tables do most certainly not make sense in their previous role as a (constructional) tool, but they do in the role of an '*independent' form of representation*: For example, tables have the characteristic expressing specific quantitative patterns more clearly and more immediately than do other forms of representation. Therefore using a table, one can 'observe' a constant relative increase by comparing the values in the table in an elementary way. (A corresponding interpretation of the algebraic expression would require knowledge about interpretation of the term structure; the function graph primarily targets qualitative patterns.)

4. Shifting of Required Basic Knowledge and Basic Skills

On the one hand, producing representations by CAS reduces the students' operative activities (and the corresponding required basic knowledge and basic skills), on the other hand, it requires (traditional as well as extended) competences from the students in other fields:

extended competences in the field of the *symbolic* (new syntactic rules; structure of algebraic expressions; hierarchy of operations, etc.)

- increased emphasis of a *functional understanding* (e. g. equations of functions are base of graphical and tabular representations; modules are functions in several variables)
- *Interpreting*: on an inner mathematical level; in context; control and reflection of representations; 'translations' between the different forms of representation. But the interpretation work in connection with operating rules is taken over by CAS.

- *(traditional) mathematical basic knowledge* is required (for the input of symbolic expressions, for the reflection and interpretation of CAS-representations in context, etc.)

5. Closing Remarks

CAS supports the use of different (forms of) representation(s) in the mathematics classroom. This should be used in an adequate didactical way for the development of basic skills in the field of representation and interpreting as well as for the development of the basic understanding of mathematical concepts and for reflections. The goal of a modern generally educating mathematics teaching cannot be to use the representational possibilities of CAS primarily for visualizing operative procedures and methods which would (and should) in any case be outsourced to CAS.

References

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