BACK

SYMBOLIC PROBLEM GENERATION IN A COMPUTER ALGEBRA SYSTEM

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Abstract
Since it is generally considered that the role of a Computer Algebra System (CAS) in education is to enable students to solve hard, real-life problems without getting bogged down in minutiae of symbolic computation, the issue of generating plain drill exercises within CAS environment is usually not viewed as important. The premise of this paper is that certain fundamental skills of basic algebra have to be mastered through repeated practice of drill exercises. Today, typical high-school algebra teachers are often faced with workbook / software combinations that do not meet their specific didactical needs. This is a surprising state of affairs, considering the power of existing CA systems. In this work, we will describe how an educational CAS can be used to generate problems that are based on teacher defined templates.

Introduction to Algebrator

Algebrator is a CA system specifically designed for high-school algebra students and teachers. Besides being able to solve a wide variety of symbolic problems, it also provides a step-by-step solution process and explanation of each transformation it performs. The system is highly customizable in respect to the details and the level of interaction with students. The following is an example of a context sensitive-explanation generated by Algebrator:

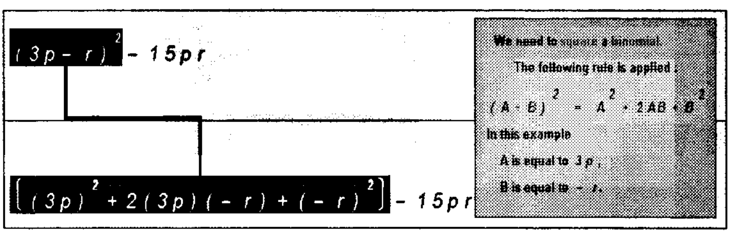

Fig. 1 – Context sensitive explanation generated by Algebrator

Template Based Problem Definition and Generation

One of the goals of Algebrator is to free the teachers from the drudgery of creating exams, homework assignments and other similar documents. Typical mathematics education software does this by either supplying a database of pre-made problems or generating them based on built-in templates. If these templates do not produce the type of problems that are needed, the teachers are faced with the prospect of manually entering problems of their own choosing. There is very often a need to create a set of similar exams or homework assignments in order to diminish the unwanted student collaboration. These are often referred to as twin documents; *twin* being a description of a practical restriction rather than of a voluntary choice. Ideally, each student should have a slightly different set of take-home problems.

The problem of generating multiple problems of similar algebraic structure is addressed in Algebrator through user-defined problem templates. A teacher can define a problem in general terms and then let the system generate a number of instances within a specified range. Here is a definition of a problem template that creates a reducible fraction in which the numerator is a difference of squares or cubes, or sum of cubes, while the denominator is one of the numerator's factors:

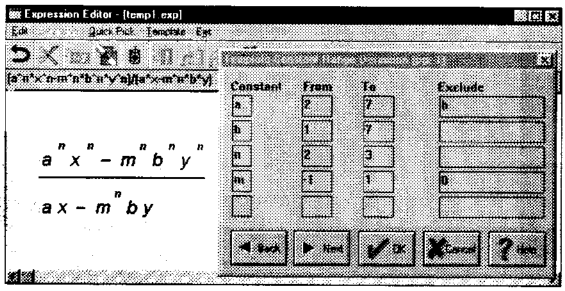

Fig. 2 - Defining a template problem

This template can be used to generate problem instances such as:

$$\frac{9x^2-16y}{3x-4y}, \frac{27x^3-216y^3}{3x-6y}, \frac{125x^3+y^3}{5x+y}$$

Some more complicated templates require a little bit of preparatory work. For example, a sum of squares is factorable when there exists a perfect square term that is also the 'missing' term in binomial expansion. For such a term to exist, the original sum of perfect squares has to be of the form $a^{4n}+2^{4m-2}b^{4p}$. In such a case factoring can be achieved by adding and subtracting the middle term $2\sqrt{a^{4n}}\sqrt{2^{4m-2}b^{4p}}$ which, when simplified, yields the perfect square $2^{2m}a^{2n}b^{2p}$. The following procedure can then be used to factor the expression:

$$\begin{aligned}
&a^{4n}+2^{4m-2}b^{4p} \pm 2^{2m}a^{2n}b^{2p} = \\
&(a^{4n}+2^{2m}a^{2n}b^{2p}+2^{4m-2}b^{4p})-2^{2m}a^{2n}b^{2p} = \\
&(a^{2n}+2^{2m-1}b^{2p})^2-(2^m a^n b^p)^2 = \\
&(a^{2n}+2^{2m-1}b^{2p}-2^m a^n b^p)(a^{2n}+2^{2m-1}b^{2p}+2^m a^n b^p)
\end{aligned}$$

The template $\dfrac{(ax+y)^{4n}+2^{4m-2}b^{4p}}{(ax+y)^{2n}+2^{2m-1}b^{2p}-2^m(ax+y)^n b^p}$ uses the reasoning above to create a set of reducible fractions such as :

$$\frac{1+4b^{12}}{1+2b^6-2b^3}, \frac{a^{12}+4b^4}{a^6+2b^2-2a^3b}, \frac{1+64b^{12}}{1+8b^6-4b^3}$$

Generating the following problem that produces the given solution:

$$\frac{4x+1}{-6x^2-13x+5}+\frac{2x-4}{10x^2+33x+20}+\frac{4x+5}{15x^2+7x-4}=-\frac{3x-5}{(3x-1)(5x+4)}$$

poses another kind of difficulty. Notice that after addition, one factor in the denominator gets reduced with a factor from a newly created factorable numerator. Here is one of the possible templates for the problem above:

$$\frac{ax+b}{cex^2+(cf+de)x+df}+\frac{gx+h}{eix^2+(ej+fi)x+df}+\frac{kx+l}{cix^2+(cj+di)x+df}=\frac{mx+n}{cix^2+(cj+di)x+df}$$

Before proceeding with the template generation we need to find out the relationship between the range variables in the problem and the solution. Simple algebraic manipulation yields the following equations:

$$
\begin{aligned}
&ai + gc + ke = me\\
&aj + bi + gd + hc + kf + le = mf + ne\\
&bj + hd + lf = nf
\end{aligned}
$$

Finding integer solution sets for this system is quite hard. Fortunately, Algebrator can generate instance problems directly from the equations:

$$
\frac{3x+2}{-2x^2+6x-4}+\frac{-x+1}{x^2-2x+1}+\frac{-4x-1}{-2x^2+6x-4},
$$

$$
\frac{4x+5}{-20x^2-13x+15}+\frac{-2x-1}{-10x^2+x+3}+\frac{2x+4}{8x^2+14x+5},
$$

$$
\frac{4x+1}{-6x^2-13x+5}+\frac{-2x+4}{-10x^2-33x-20}+\frac{4x+5}{15x^2+7x-4}
$$

Based on methods described above, Algebrator can also generate multiple choice exams (together with correct and incorrect solutions), homework assignments and entire problem workbooks.

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