BACK

HISTORICAL CURVES AND NEW TECHNOLOGY

Manfred KRONFELLNER, Vienna, Austria
Vienna University of Technology
Institute of Algebra and Computational Mathematics
A-1040 Vienna, Wiedner Hauptstraße 8-10
m.kronfellner@tuwien.ac.at

Abstract
In order to solve problems (e.g. the 3 classical problems) the ancient Greeks invented special curves (conchoid, cissoid, quadratrix, Archimedian spiral). These curves, which are rather complicated to draw by hand, can easily be demonstrated with the help of new technologies. Hence the opportunity comes up to reanimate these interesting objects in mathematics education and to discuss their historical background. With technology we can also show the advantages of other coordinate systems beside the Cartesian system (and the - overemphasized - graphs of functions). When implementing these curves into classroom teaching an important question arises: How to pose related exam questions?

Unbalances in traditional mathematics education

According to Roland Fischer (cf. Werner Peschek's contribution in this volume) mathematics consists of

Representation + Operation + Interpretation

In traditional mathematics education the component "Operation" is highly overemphasized whereas "Representation" and "Interpretation" are neglected or suppressed. As a consequence of the increasing availability of new technology in schools we should reduce "Operation" and spend more attention to "Representation" and "Interpretation".

Beside this asymmetry there are other fields in school mathematics in which we can find overemphasized and neglected parts. Cartesian graphs of functions are such a dominant field whereas implicit plots, parametric plots, polar plots and recursive representations are of minor importance in traditional mathematics education.

Another asymmetry I see in the fact that most contents of usual school mathematics have been developed between the 16th and the middle of the 19th century. Mathematics from the antiquity or mathematics from the 20th century only play a marginal role.

In order to draw an appropriate picture of mathematics as a highly applicable field on the one hand and as a cultural phenomenon on the other we should try to avoid such asymmetries and to support underdeveloped parts. For there are a lot of such parts we have to think about possible synergies by covering with each content more than only one goal.

New technology and historical curves

In their various attempts to solve the three classical problems of the antiquity (duplication of the cube, trisection of an angle, quadrature of a circle) the ancient Greek mathematicians invented not only mechanical devices but also different curves: cissoid, conchoid, quadratrix, Archimedian spiral. (KRONFELLNER 1998, p. 96ff) This topic is a good opportunity to talk about mathematics as a cultural heritage and about the importance of these problems for the development of mathematics for more than 2000 years (esp. the axiomatic method; cf. KRONFELLNER 2000). On the other hand these historical curves can demonstrate the restrictions when using graphs of functions in a Cartesian coordinate system and the advantages of other representations such as implicit, parametric or polar plot which in addition suggest the use of computer algebra systems. In other words this topic offers the opportunity for such a synergy mentioned above, namely between history of mathematics and modern technology – for some people perhaps a surprising connection.

The exam situation

The exam situation is a second point where new technology on the one hand or history of mathematics on the other hand lead to analogous problems:

Is technology also available/allowed in the exam situation?

Should history of mathematics be taken into account in assessment?

If yes: How?

If no: Why should students learn it (beyond intrinsic interest)? They (and their parents) can regard these parts of teaching as vasted time. In their opinion the teacher should practise what he/she intends to examine. In other words: he/she should concentrate on what is "important".

> *Proposition 1:* *A content X is important in mathematics teaching if and only if X is taken into account in the exam situation.*

> *Proposition 2 (Main theorem of traditional math teaching):*
> *An exam question/problem is a question/problem which can be practised by using a plantation of similar/analogous (but not identical) problems.*

> *Corollary: An important content in (traditional) math education is a content which can be practised by using plantations of similar problems.*

This – of course! – is only an **ideology** (or "teacher belief") but a widespread one. This ideology is one of the main obstacles for innovations in mathematics education. A couple of analogous problems is – using new technology – no longer a couple of several problems but only one single problem. And now the question arises: How to examine this knowledge (according to proposition 2)?

The same difficulty occurs in connection with history of mathematics: There is only one Archimedian spiral, there is only one idea to use it for the quadrature of a circle. To pose in this special case several analogous but not identical problems would not make sense.

Therefore I propose to change the ideology radically: I propose to examine – sometimes! - also facts and problems identical to those taught and explained in the lessons. These could be much more demanding than series of tedious straightforward calculations.

References

- KRONFELLNER, M.: Historische Aspekte im Mathematikunterricht, Hölder-Pichler-Tempsky, Wien 1998
- KRONFELLNER, M.: The Duplication of the Cube, in: FAUVEL, J., van MAANEN, J.: History in Mathematics Education. An ICMI Study. Kluwer Academic Publishers, Dordrecht 2000