BACK

THE MATHEMATICS FINAL EXAM QUESTIONS FOR THE ITALIAN EXPERIMENTAL SCIENTIFIC LICEO DISCUSSED AND SOLVED WITH DERIVE 5

Rocco FAZIO
Liceo scientifico "R. Canudo" – Gioia del Colle (BA) Italy
rofaz@tin.it

Abstract
Since 1985 italian students of almost all the secondary courses can attend an experimental course of Mathematics and Physics called P.N.I. (Piano Nazionale per l'Informatica, in English National Plan for Computer Scienze). The goal of this experimental curriculum is the reform of contents and methods of mathematics teaching and learning and the introduction of the study of algorhitms and computer programming (expecially Pascal language) in the secondary mathematical curriculum. The last final exam questions are discussed and solved with Derive 5.

Here is a short summary of mathematics curriculum for P.N.I. courses: arithmetic, algebra, linear algebra, euclidean and analytic geometry, trigonometry, logarithmic and exponential functions, statistics and probability, differential calculus and numerical analysis, mathematical logic, algorhitms and computer science.

And here are some skills, expected in this experimental curriculum, that can be well tested, in my opinion, in a CAS environment like Derive 5: working with mathematical symbolism and beeing able to recognize the syntactical rules of formulas manipulations; dealing with various problematic situations using mathematical methods, suitable for their rapresentation; building solving procedures and translating them in computer programs, if appropriate. However, during the final exam, students were allowed to use only a not programmable calculator (I think it's the same as taking a driving licence exam by a bicycle). Students had five hours for solving two of the following three problems.

PROBLEM N. 1

Let $f(x)$ be a real function of a real variable, satisfying the following conditions: $f(x_0)>0$, $f'(x_0)>0$, $f''(x_0)=0$, with $x_0\in\mathbb{R}$.

a) Explain why such conditions are not sufficient to decide the behaviour of $f(x)$ in a neighbourhood of $x_0\in\mathbb{R}$.

b) Find at least three polinomial functions $f(x)$, with degree greater than 1, with different behaviours at x_0 and such that $f(0)=1$, $f'(0)=1$, $f''(0)=0$.

c) Find, if possible, all the tangent lines to the graphics of the found functions, parallel to the line $y=x+1$.

d) Demonstrate the formula of the first derivative of the function $f(x)=x^n$, $n\in\mathrm{N}^*$.

COMMENTS

In the following table I explain my ideas regarding to the use of technology in proposing and solving the 4 questions of the problem:

Question	Technology
a)	No
b)	Yes/No
c)	Yes
d)	No

Question a) doesn't need the use of technology; students have to remember the theory of flex points and expose it.

Question b) may be solved both without technology and with it.

Students have to observe that, for a polinomial function of degree *n*

$$f(x)=a_0x^n+a_1x^{n-1}+\ldots+a_{n-1}x+a_n,$$

is $f(0)=a_n$, $f'(0)=a_{n-1}$, $f''(0)=a_{n-2}$, and so, to satisfy the conditions b) of the problem, it is to be satisfied $a_n=1$, $a_{n-1}=1$, $a_{n-2}=0$.

Then they have to choose at least three polinomial functions with different behaviours at $x_0=0$ satisfying b). A possible choise may be:

$$f(x)=x^3+x+1\ ,g(x)=x^4+x+1,\ h(x)=x^5-x^3+x+1$$

The graphs of the functions are not required; in a technological environment, however, students can also show their skill in defining, representing and investigating the properties of family of curves like $y = k \cdot x^3 + x + 1$ or $y = h \cdot x^4 + k \cdot x^3 + x + 1$ $(h, k \in \mathbb{R})$, for example.

Question c) may be very difficult in a paper and pencil environment, according to the choice of the functions; students have to solve the equations $f'(x) = 1$, $g'(x) = 1$ and $h'(x) = 1$. They don't know, *a priori*, if such equations will be solved algebraically or numerically. The text of the problem is " if possible"; but what does it mean? Perhaps it means "if they exist"; in this case equations can be solved also numerically. In a paper and pencil environment it is very laborious solve numerically an equation; in a CAS environment like Derive 5 students can not only solve equations numerically in a very easy way, but they can also write and execute simple programs like this:

```
bisezione(a, b, error) :=
Prog
Loop
c := (a + b)/2
If ABS(b - a) < error ∨ f(c) = 0
RETURN c
If f(c)·f(a) < 0
b := c
a := c
```

If the request of realizing a program had been given, the exam would have tested an other skill.

Question d) No comment is made on this question.

PROBLEM N. 2

In a plane Oxy consider the points $A(0;2)$, $B(1;1)$ e $C(1;0)$.

a) Find the equation of the cirle γ inscribed in the triangle OAB.
b) Find the equation of the affine transformation τ with fixed points O and C and with $\tau(B) = A$.
c) Calculate the area of the triangle CAA', where is $\tau(A) = A'$.

d) Find all the fixed points and all the fixed lines of τ.
e) What lines, among the fixed ones, are tangent or external to γ?

COMMENTS

This is a very simple question; I think it can be proposed both with technology and without it. Solving it with a CAS like Derive 5 is not quite immediate, and requires some further skills: to be able to choice the data structure for representing points, for example.

Students could solve question e) in two ways:

- with a lot of calculations (solving a system of equations and then a second degree disequation)
- without almost any calculation (with a simple geometrical reasoning).

I'm sure that a great many of them, working during the exam with paper and pencil, choose the first way of solving the question.

PROBLEM N. 3

Consider the function $f(x) = a\ln^2 x + b\ln x$.

a) Find the values of a and b for which $f(x)$ has a local minimum in $\left(\sqrt{e}, -\frac{1}{4}\right)$;
b) Graph the curve $y = f(x)$ with the values found in a) and calculate the area of the finite region between the curve and the x-axis.

COMMENTS

This problem requires only some calculation skill, in particular in solving an integral by parts. In a CAS environment like Derive 5 it is very simple to solve it.

The last question is very simple and it isn't anyway connected to the rest of the problem.

Derive 5 files are available in Internet: `www.campustore.it/didattica`