BACK

ASSESSMENT IN THE CAS AGE: AN IRISH PERSPECTIVE

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Abstract
The purpose of this study is to analyse how the availability of Computer Algebra Systems (CAS) would affect the Leaving Certificate mathematics papers (the Leaving Certificate is a terminal second level examination in the Republic of Ireland). The most recent higher level mathematics papers are analysed in the following manner: the questions are answered with the help of DERIVE; a classification of the types of question is then made; fundamental questions arise. The idea of a CAS-index for examinations is introduced.

Project

This project had the following objectives: answer the 1999 Leaving Certificate Higher Level Mathematics papers with the aid of DERIVE; classify types of question with respect to DERIVE; investigate the implications; devise an index for assessment in the CAS age. The following restrictions were applied: use only a CAS (DERIVE); use DERIVE only in a basic manner (no user programming; no auxiliary programs). The rationale for the second restriction was to try to use the level of facility with DERIVE that an average student would have after an introductory course. The first restriction is more problematic: it could be argued that we should also allow the use of geometry packages, statistics packages, group theory packages etc. The trouble with this approach is where does one stop? So a CAS was chosen as the most powerful of these.

Leaving Certificate Mathematics, Higher Level

This examination consists of two papers, the main topics being as follows:

Paper 1: algebra; complex numbers; series; induction; calculus

Paper 2: geometry; vectors; trigonometry; probability and statistics; optional topic

Classification of questions

The questions, which were written for a CAS-free enviroment, were classified according to the following 4 categories: trivial with CAS; easy with CAS; difficult with CAS; CAS-proof. This is a rather simple one-dimensional classification scheme. More elaborate schemes have been proposed by Vlasta Kokol-Voljc (also one-dimensional) and by Bernhard Kutzler (two-dimensional). Trivial with CAS: typically, these questions reduce down to 2 or 3 steps, such as: enter expression and solve; enter expression and integrate. An example of this type of question is Paper 1 question 7(a) [see appendix]. This involves the technique of differentiation. Easy with CAS: in this category the student needs to know how to answer the question but DERIVE reduces the level of difficulty substantially. An example is Paper 1 question 7(b)(i) [see appendix]. The student needs to know the Chain Rule: DERIVE does the calculations. Difficult with CAS: these questions remain difficult though DERIVE is of some help. An example is Paper 1 question 7(c) [see appendix]. This is a good example of a question which is useful in a CAS-environment. It involves a cubic equation containing a parameter; so it involves a family of curves. This would be difficult to attack graphically using only pencil and paper, so an analytic solution would probably be given. Using CAS either method could be used; the graphical method gives a very intuitive answer. CAS-proof: in this category DERIVE is of no use or of minimal use. These questions may ask the student to answer in a specified manner. An example is Paper 1 question 6(b) [see appendix]. This involves differentiation from first principles.

Results

We obtained the following results:

	Paper 1	Paper 2
Trivial	48 %	9 %
Easy	27 %	6 %
Difficult	8 %	0 %
CAS-proof	17 %	85 %

CAS Index

This is an attempt to quantify how suitable mathematics examination papers are for the CAS age. We wish to measure the following: for papers designed for a CAS-free environment: how suitable are they for a CAS-supported environment? (One might also argue that a second index is needed: for papers designed for a CAS-supported environment: how suitable are they? This second question is not explored here.) Of what use would such a CAS index be? Firstly, some examiners may not be aware of the power of CAS; the index would give some measure of this. Secondly, if CAS is not allowed in an examination but scientific calculators are allowed, problems of enforcement can arise. The CAS index gives a measure of the advantage to a student of using CAS when it is not allowed.

CAS Index: calculation

We assign weights to the 4 categories. Proposed weights: trivial:0; easy:0; difficult:1; CAS-proof:1. The interpretation is that questions which are trivial or easy with CAS are no longer suitable in the CAS age. Arguments could be made for different weights, e.g. 0;1/3;1;1. However, the proposed weights have the virtue of simplicity. The index ranges from 0 to 10. 0: unsuitable in the CAS age; 10: suitable in the CAS age. Let x% = student's score answering only questions which are trivial or easy with CAS. Convert as follows: (100-x)/10 (rounded to a whole number).

Paper 1

75% of the questions are trivial or easy with CAS. However, because of the rubric of the examination paper (answer 6 out of 8 questions), a student would score 81% answering only trivial or easy questions. Convert to the index: (100-81)/10 = 2 (rounded). Paper 1 scores 2. Conclusion: unsuitable in the CAS age.

Paper 2

15% of the questions are trivial or easy with CAS. A student would score 28% answering trivial or easy questions. Conversion: (100-28)/10 = 7. Paper 2 scores 7. Conclusion: still reasonably suitable in the CAS age.

Appendix

6 (b) Find from first principles the derivative of sin x with respect to x.

7 (a) Find the derivative of SQRT(x^2+1).

(b)

(i) Let x = t - sin t cos t and y = 4 cos t, 0 < t < π/2. Show that dy/dx = -2/sin t

(ii) Find the slope of the tangent to the curve x^2 - y^2 - x = 1 at the point (2,1).

(c) Let f(x) = x^3 + k x^2 - 4, x a real number and k > 0. Show that the coordinates of the local minimum and local maximum of f(x) are (0,-4) and (-2k/3,(4k^2-108)/27) respectively. Find,

(i) the range of values of k for which f(x) = 0 has three real roots

(ii) the value of k for which f(x) = 0 has three roots, two of which are equal.

[The Leaving Certificate; Higher level mathematics, Paper 1, 1999]